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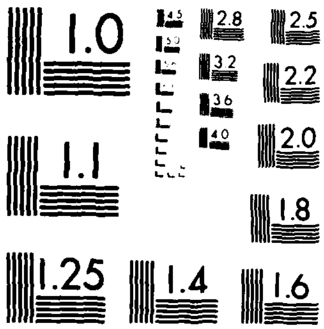

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THE COLD FISH WAR: LONG-TERM COMPETITION IN A DYNAMIC GAME

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The Cold Fish War: long-term competition in a dynamic game

I. Introduction

In games where players move several times, it is reasonable that their choices at any given moment should depend on information about the previous history of play. This dependence has been exploited to characterize the equilibrium behavior of infinitely-repeated games [supergames]. This paper applies the methods of analysis developed for the supergame to an example of a more general class of games called dynamic games.

The "Great Fish War" model constructed by Levhari and Mirman [1980] provides an interesting and tractable example of a dynamic game. It concerns the joint exploitation of a renewable resource by several players. In their paper, Levhari and Mirman apply a Cournot-Nash solution concept that satisfies an additional "backwards induction" rationality constraint. In order to do this, they first consider a situation in which there is but a single future period. Assuming equal division in this last period, they construct the payoff functions for the penultimate period, and solve for the Nash equilibrium levels of extraction as a function of the initial stock. Repeating the process, they solve for equilibrium behavior in each previous period. The sequence of strategies converges to a limit as the induction proceeds, and this limit determines a steady-state equilibrium.

The essence of the recursion had been brought out by Mirman [1979], which formulated the decision problem faced by each player. The extraction at any given instant was chosen to maximize the value of current consumption plus the expected present discounted value of the remaining stock. Denoting the extraction by two players as c(1) and c(2), the value of current consumption by player i as U(i,c(i)), the discount factor characterising player i as δ_i, and the renewal function by f, where f(x) is the stock next period if x was the stock at the end of the present period, the value to player i of a stock S, V(i,S), satisfies:

$$(1) \qquad V(i,S) = \max_{c(i)} \; U(i,c(i)) + \delta_i V(i,f(S-c(1)-c(2)))$$

This is essentially the same as the backwards induction solution: both depend on previous history only through the current stock, and if backwards induction converges, it will converge to a solution to (1). Therefore, both approaches yield an equilibrium in "memoryless" strategies.

These papers left open two outstanding questions, which have implications beyond the specific example. The first is that a single externality links the two players; that created by the renewal process. If the resource were to be sold in a noncompetitive market, or used in the production of commodities sold on such a market, additional externalities would be introduced [Levhari and Mirman, op. cit. note 3]. These have been analyzed by Reinganum and Stokey [1981].

In addition, with either the "heuristic" procedure or backwards induction

current consumption depends only on current stock, and is optimal at every stock level. Other strategies might be considered which contain explicit or implicit threats [Levhari and Mirman, op. cit. page 323]. However, as in the repeated-game case, simple forms of dependence can add a great deal to our understanding of equilibrium behavior. Indeed, a recent paper by Lewis and Cowens [1983] exploits such strategies to determine the conditions under which the symmetric cooperative duopoly outcome can be supported by a perfect equilibrium.

The strategies we shall consider are obtained under relaxed optimality conditions, and involve more explicit dependance on the past, albeit of a particularly simple kind. Typically, strategies will consist of a sequence of extractions to be followed if no player deviates from prescribed behavior, together with a "punishment" strategy to be employed if there have been defections. Weak global optimality, which requires that each player's strategy be a best response to the other player's, defines Nash equilibrium. Local optimality, which requires that every player's choices in every conceiveable contingency be weakly globally optimal, defines subgame perfect equilibrium.

This paper extends the analysis of "cooperative equilibria" of the Levhari-Mirman example by using methods developed for repeated games to study equilibria which Pareto-dominate the Cournot-Nash equilibrium of Levhari-Mirman. The term "Cold War" reflects the fact that such "cooperative" behavior is supported by threats which are not carried out. The first section summarizes the repeated game tools and the results obtained with them. The next section contains the Levhari-Mirman example, and characterizes the "cooperative" equilibrium outcomes. The third section describes a sufficient condition for subgame perfection of an equilibrium outcome and demonstrates that backwards induction optimality is not implied by perfection, and that there are many desireable perfect equilibria that cannot be constructed by backwards induction. The fourth section describes the perfect equilibria in the nonrenewable or exhaustible resource case. The last section contains a discussion of some limitations and possible extensions.

II. Equilibrium and Perfect Equilibrium in Repeated Games

First, let us consider a Prisoners' Dilemma game repeated twice. Players are informed of the outcome of the first round before they make their final choices, and each maximizes the sum of his payoffs. The one-shot game is:

	H	G
H	(3,3)	(0,4)
G	(4,0)	(1,1)

Figure 1

Applying backwards induction to this game (or any game with but a single

one-shot equilibrium) we should expect players to use their one-shot equilibrium strategies (here, (G,G)) in the last period, regardless of history. The payoff function for the first period is:

(4,4)	(1,5)
(5,1)	(2,2)

Figure 2

and we conclude that any memoryless sequence of equilibria of a one-shot game is a backwards induction equilibrium of the finite repetition of that game. However, there are other equilibria. Suppose that each player decides to play H in the last game iff (H,H) was played in the first game, and to play G otherwise. The first-stage payoff function is:

(6,6)	(1,5)
(5,1)	(2,2)

Figure 3

This game has 3 equilibrium points: (H,H), (G,G) and a symmetric mixed-strategy equilibrium. Of course, the expectation of (H,H) in the second period would probably not be fulfilled unless the players could credibly commit themselves to it, but from the first-period point of view it would still be an equilibrium.

If the one-shot game has several equilibrium points, then backwards induction does not eliminate dependance on the past. Consider the two-fold repetition of Battle of the Sexes:

	L	R
T	(3,6)	(0,0)
B	(0,0)	(6,3)

Figure 4

This game has three equilibria in the last period: if p (resp. q) is the probability of player I (II) using strategy T (L), then the one-shot game has equilibria at (p,q) = (0,0), (1/3,2/3), and (1,1), with payoffs (3,6), (2,2), and (6,3). Any of these can credibly be used in the last period without violating backwards induction rationality. Depending on the way in which these are "assigned" to outcomes of the first round, a variety of equilibria can result. If the players decide on a single equilibrium to use in the last round, no matter what the previous history, then the first-round game will differ by a constant from the one-shot game and will have the same equilibrium strategies. On the other hand, suppose that the

players agree to follow (1,1) with (1,1); (0,0) with (1/3,2/3) and anything else with (0,0). The resulting payoffs are:

(6,12)	(6,3)
(6,3)	(8,5)

Figure 5

Figure 6 shows the set of backwards induction equilibrium average payoffs for the 2-stage game.

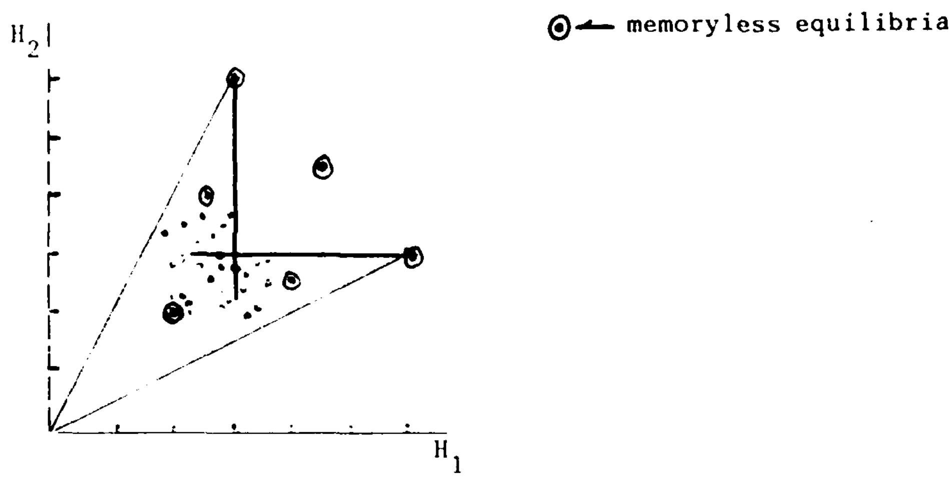

Figure 6

In the undiscounted infinitely-repeated game, the limiting average payoff may not exist. However, we can restate the Nash equilibrium requirement as: a player will not defect if the lim inf of his equilibrium average payoffs exceeds the lim sup his average payoffs obtained by a unilateral change of strategy. With this definition, all equilibrium sequences converge, and we can use the "grim" punishments: any defection is to be punished by holding the defector to his minmax payoff in every future round. This leads to the Folk Theorem [cf Aumann, 1976]: every payoff that is feasible (in the convex hull of the one-shot payoffs) and individually-rational (gives each player at least his minmax payoff) is the payoff to an equilibrium of the undiscounted repeated game, and vice versa.

If future payoffs are discounted, the immediate payoff to a defection may become more important than the eventual punishment, so the equilibrium set cannot be described in terms of payoffs alone.

A subgame perfect equilibrium is one in which the chosen strategies induce equilibrium behavior in every subgame. In repeated games, subgames are defined by histories of play in previous rounds. In the undiscounted infinitely-repeated game, the set of payoffs to subgame perfect equilibria

is the same as the set of equilibrium payoffs. To see this, it is sufficient to define a sequence of "punishment limits" e(t) that tends to 0. If a player defects from the equilibrium strategy behavior at round t, he will be punished by repeatedly being held to his minmax payoff until his cumulative average payoff exceeds his minmax payoff by no more than e(t). At this point, play returns to the cooperative sequence. Whatever residual profit the player may have disappears in the limit, unless he defects again, in which case he faces a longer punishment, etc. If another player fails to punish in the appropriate fashion, then that player becomes the defector.

This result also fails to carry over to the discounted game. In some cases, e.g., when there is an equilibrium of the one-shot game giving the players their minmax payoffs (Prisoners' Dilemma, Battle of the Sexes, etc.), the grim punishment of a defector is itself an equilibrium, so the sets of equilibrium and perfect equilibrium payoffs are identical [Cave, 1979], but there are many examples where this is not the case.

There are other notions of perfection that are stronger than subgame perfectness. Selten [1975] has defined the concept of trembling-hand perfection, which requires the existence of a sequence of completely-mixed strategies converging to the equilibrium strategies, with the property that no inferior strategy of any player is given more than asymptotically 0 weight. In two player games, or games where each player's payoff depends only on the aggregate of the other players' strategies, this requirement is equivalent to the simpler condition that no player uses a weakly dominated strategy with positive probability. That this is stronger than subgame perfection can be seen in the example shown in Fig. 5, where only the equilibrium (0,0) with payoff (8,5) is trembling-hand perfect, although all equilibria are subgame perfect by construction.

In the dynamic setting, one cannot make reference to the one-shot game in the same way, but there are certain similarities. Strategies can still be divided into specified behavior (the choices that follow histories that occur with positive probability) and "punishments" (the rest of the choices). We can specify a sequence of moves in each period and check to see whether there exists a punishment that will make players adhere to this sequence. For subgame perfect equilibria, we require that the punishment sequence itself be an equilibrium. However, instead of using a repeated one-shot equilibrium for punishing defections, we shall use the Nash-Cournot equilibrium obtained by either backwards induction or "heuristic" methods.

II. Equilibrium in an example of a dynamic game

This example is drawn from Levhari and Mirman (op.cit., Section 2). There is an initial stock of fish S(0). If the stock at the beginning of period t is S(t) and nation i (= 1,2) extracts the fraction h(i), where h(1) + h(2) is ≤ 1, the stock at the beginning of period t+1 will be:

(2) $$S(t+1) = [(1-h(1)-h(2))S(t)]^{\alpha}$$

for some $\alpha \in [0,1]$. Each player has the utility function $U(i,c(i)) = \ln(c(i))$ and discounts the future using the discount factor $\delta_i \in [0,1]$. By virtue of the logarithmic utilities and exponential growth and discounting, we shall limit our attention to constant cooperative sequences, where each country in each period consumes a fixed proportion of the available stock.

If a nation defects, by consuming or extracting a different proportion of the available stock, this will become apparant during the next period (the growth law is deterministic) and can be punished then.

If the constant fractions $h^*(1)$ and $h^*(2)$ are used, the time paths of stock and consumption are given by:

$$(3) \qquad S(t) = [1-h^*(1)-h^*(2)]^{g(t)} S(0)^{\alpha^t}$$

and

$$(4) \qquad c(i,t) = h^*(i)S(t)$$

where

$$(5) \qquad g(t) = \frac{\alpha(1-\alpha^t)}{1-\alpha}$$

The present discounted value of using the strategies h^* is

$$(6) \qquad V(i,h^*) = \frac{\ln(h^*(i))}{1-\delta_i} + \frac{\ln(S(0))}{1-\alpha\delta_i} + \frac{\alpha\delta_i \ln(1-h^*(1)-h^*(2))}{(1-\delta_i)\ (1-\alpha\delta_i)}$$

which we can simplify by multiplying through by $(1-\delta_i)\ (1-\alpha\delta_i)$

The strongest punishment that can be inflicted is immediate consumption of the remaining stock of fish. The payoff to both players if this is done is $-\infty$ (provided $\delta_i > 0$); hence any h^* is an equilibrium h^*.

III. Perfect Equilibrium

In this section, we shall find the h^* which can be sustained when defections are met with an immediate and permanent return to the Cournot-Nash extraction path; this will give us a subset of the perfect equilibrium payoffs.

First, we can use the payoff function (6) above to determine the Cournot-Nash equilibrium. The best-response functions are:

$$(7) \qquad h^b(i) = (1-\alpha\delta_i)(1-h(j)) \equiv (1-\beta_i)(1-h(j))$$

and the Nash Equilibrium extractions are:

$$(8) \qquad h^n(i) = \frac{(1-\beta_i)\beta_j}{\beta_i+\beta_j-\beta_i\beta_j}$$

The value of noncooperative play starting with a stock of S is therefore:

$$(9) \quad (1-\delta_i)(1-\beta_i)V^n(i,S) = (1-\beta_i)\ln(1-\beta_i) + \beta_i\ln(\beta_i) + \ln(\beta_j) - \ln[\beta_1+\beta_2-\beta_1\beta_2] + (1-\delta_i)\ln(S)$$

To find the best defection from $h^*(1), h^*(2)$ possible when the stock is S, and defections are punished by a return to noncooperative play, player i solves:

$$(10) \qquad \max_h \ln(hS) + \delta_i[V^n(i,S^\alpha(1-h-h^*(j))^\alpha)]$$

The best defection is the same as the best defection against stationary play of $h^*(j)$:

$$(11) \qquad h^b(h^*(j)) = (1-\beta_i)(1-h^*(j))$$

The payoff that i can anticipate if he makes this best defection is:

$$[(1-\beta_i)\ln(1-\beta_i)+\beta_i\ln(\beta_i)+\delta_i(\ln(\beta_j)-\ln(\beta_1+\beta_2-\beta_1\beta_2))$$

$$(12) \qquad +(1-\delta_i)(\ln(S)+\ln(1-h^*(j)))]/[(1-\delta_i)(1-\beta_i)]$$

We can now write the perfect equilibrium condition. Defining K(i) to be the expression

$$(1-\beta_i)\ln(1-\beta_i)+\beta_i\ln(\beta_i)+\delta_i(\ln(\beta_j)-\ln(\beta_1+\beta_2-\beta_1\beta_2))$$

we have

$$(13) \quad (1-\beta_i)\ln(h^*(i)) + \beta_i\ln(1-h^*(1)-h^*(2)) - (1-\delta_i)\ln(1-h^*(j)) \geq K(i)$$

As a check, it will be noted that the Cournot-Nash Equilibrium shares h^n satisfy these constraints with equality.

Now fix the total proportion extracted at h^* and use $h^*(1) + h^*(2) = h^*$ to rewrite (13) as:

$$(14) \qquad f(h^*(1), h^*) \geq 0$$

and

$$(15) \qquad f(h^*-h^*(1), h^*) \geq 0$$

These are monotone increasing and decreasing functions of $h^*(1)$, respectively, so that (14) provides the lower bound on perfect equilibrium

$h^*(1)$, while (15) provides the upper bound. Figure 7 shows these functions.

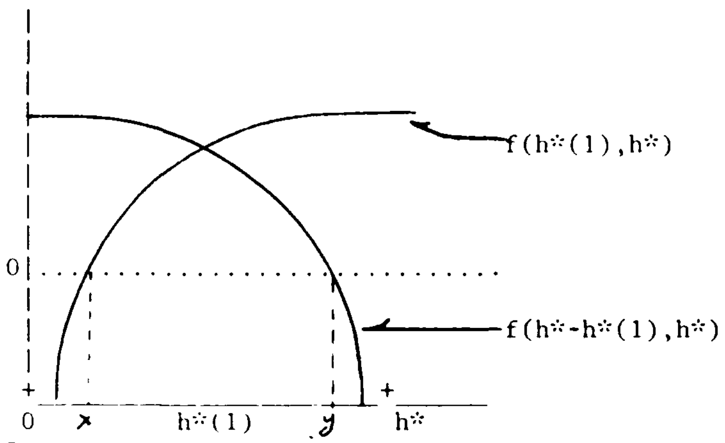

Figure 7

We shall now examine the responsiveness of the upper and lower bounds, which we denote $y(h^*)$ and $x(h^*)$, respectively, to changes in h^*

The total derivative of (14) with respect to x and h^* can be set equal to 0 to give (for i = 1):

$$\text{(16)}\qquad \left[\frac{(1-\beta_1)}{x} - \frac{(1-\delta_1)}{1-h^*+x}\right] dx = \left[\frac{\beta_i}{1-h^*} - \frac{(1-\delta_1)}{1-h^*+x}\right] dh^*$$

The coefficient of dx is positive, while that of dh^* varies with the values of h^* and x: $x(h^*)$ increases with h^* iff:

$$\text{(17)}\qquad x(h^*) > \frac{(1-\delta_1-\beta_1)(1-h^*)}{\beta_1} = x^*$$

Since f is monotone increasing, $x(h^*)$ is increasing in h^* iff

$$\text{(18)}\qquad f(x^*,h^*) < f(x(h^*),h^*) = 0$$

This will be the case iff h^* is at least h^{**}, where

$$\text{(19)}\qquad h^{**} = \exp\left[\left[\frac{1-\beta_1}{\delta_1}\right]\ln\left[\frac{1-\beta_1}{1-\delta_1-\beta_1}\right] + \left[\frac{1-\delta_1}{\delta_1}\right]\ln[1-\delta_1] + \ln[\beta_1+\beta_2-\beta_1\beta_2] + \ln(\beta_1) - \ln(\beta_2)\right]$$

We can do the same thing with $y(h^*)$: total differentiation gives:

(20) $$\left[\frac{1-\delta_2}{1-y} - \frac{1-\beta_2}{h^*-y}\right] dy = \left[\frac{\beta_2}{1-h^*} - \frac{1-\beta_2}{h^*-y}\right] dh^*$$

The coefficient of dy is negative, so that $y(h^*)$ increases with h^* iff:

(21) $$y > y^* = 1 - (1-h^*)/\beta_2$$

Again exploiting the monotonicity of f, this means that $y(h^*)$ will increase with h^* iff

(22) $$0 = f(h^*-y(h^*),h^*) < f(h^*-y^*,h^*)$$

This will be the case iff $h^* < h^\circ$, where

(23) $$h^\circ = 1 - \frac{\beta_2[\beta_1+\beta_2-\beta_1\beta_2]}{\beta_1}$$

The set of equilibrium shares is shown in Fig. 8.

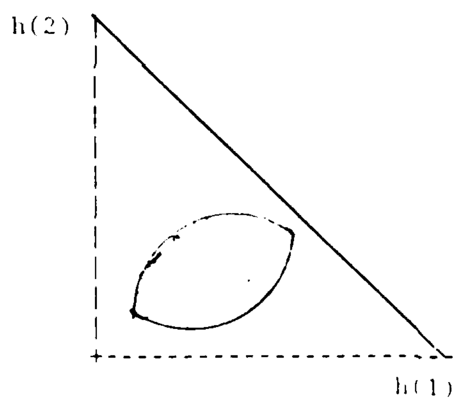

Figure 8

A particular total level of extraction can be sustained by a perfect equilibrium of this simple form iff there exists x such that

(24) $$f(z,h) = f(h-z,h) \geq 0$$

Monotonicity of f implies that there is a unique z satisfying the equation in (24), which we denote by z(h). Substituting z(h), we get

(25) $$G(h) = f(z(h),h) = f(h-z(h),h) \geq 0$$

From this it is evident that $G(0) = G(1) = -\infty$. Moreover, the existence of Cournot-Nash equilibrium shares shows that (25) is not vaccuous. G can also be shown to be concave in h, from which we conclude that the set of

sustainable total extractions is convex. While this is complicated in the general case, G becomes much more tractable if we assume that both agents have the same discount factor, δ. We can also define β to be the common value $\beta = \beta_i$. In this case, $z(h)$ is $h/2$, and G can be written:

$$(26) \qquad G(h) = (1-\beta)\ln(h) + \beta\ln(1-h) - (1-\delta)\ln(2-h) - (1-\beta)\ln(1-\beta) - \beta\ln(\beta) + \delta\ln(2-\beta) - (\delta-\beta)\ln(2)$$

In this case, the concavity of G is easy to verify:

$$(27) \qquad G''(h) = -\left[\frac{1-\beta}{h^2} + \frac{\beta}{(1-h)^2} + \frac{1-\delta}{(2-h)^2}\right] < 0$$

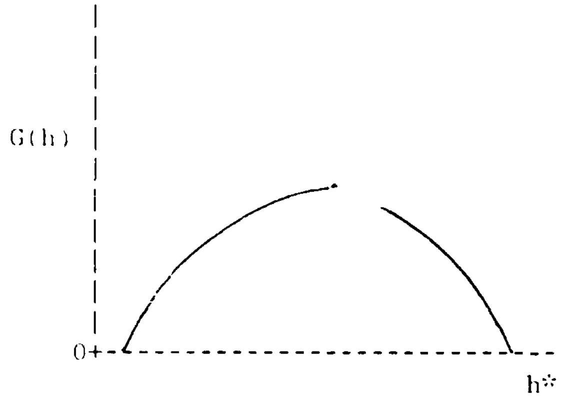

Figure 9

We can also do some limited comparative statics, to see the effect of changes in the parameters on the set of sustainable total extraction rates.

$$(28) \qquad \frac{\partial G}{\partial \delta} = \frac{[\ln(2-h)+\ln(1-\beta)-\ln(h)]}{\delta} - \frac{\beta}{2-\beta}$$

which is positive iff:

$$(29) \qquad h < 2/(R+1)$$

where

$$(29') \qquad \ln(R) = (\beta\delta)/(2-\beta) - \ln(1-\beta)$$

On the other hand, the sign of the coefficient of dh is the same as the sign of

$$(30) \qquad (1-\beta) - (2-\beta+\delta)h + \delta h^2$$

which follows the pattern +,-,+ as h increases. If the coefficients are of

the same sign at the maximal extraction rate and of opposite signs at the minimal extraction rate, then the set of sustainable extractions increases with increases in δ. This is what we would expect from the analogy with repeated games, but the dynamic game is complicated by the renewal effect. At the maximal rate of extraction, each player is just kept from increasing his catch by the future reward. If the players become more far-sighted (δ increases), the value of this reward increases. On the other hand, if δ falls to 0, only h=1 is sustainable, and it is sustainable with any shares! This does not look like the repeated-game situation where increasing myopia shrinks the set of equilibria to the one-shot equilibria. In this example, we would have to solve for the maximal and minimal sustainable extraction rates explicitly as functions of α and δ from:

$$(31) \qquad h^{1-\beta}(1-h)^{\beta}(2-h)^{\delta-1} = (1-\beta)^{1-\beta}\beta^{\beta}2^{\delta-\beta}(2-\beta)^{-\delta}$$

We shall defer further comparative statics analysis to the exhaustible resource discussion in Section IV.

We now turn to the question of the sustainability of Pareto Optimal levels of extraction.

In the symmetric case, the Pareto Optimal levels are found by solving:

$$(32) \quad \max\ wV(1,h(1),h(2)) + (1-w)V(2,h(1),h(2)) \text{ subject to}$$

$$h(i) \geq 0, \text{ each } i, \text{ and}$$

$$h(1) + h(2) \leq 1$$

for fixed $w \in (0,1)$. The logarithmic form of the V(i) ensures that the constraints will not be binding at the optimum. The first-order conditions are:

$$(33) \quad w\left[\frac{1-\beta}{h(1)}\right] = \left[\frac{\beta}{1-h(1)-h(2)}\right]$$

$$(34) \quad (1-w)\left[\frac{1-\beta}{h(2)}\right] = \left[\frac{\beta}{1-h(1)-h(2)}\right]$$

These can be solved for interior Pareto Optima:

$$(35) \quad \begin{aligned} h(1) &= w(1-\beta) \\ h(2) &= (1-w)(1-\beta) \end{aligned}$$

and the Pareto Optimal shares are those summing to $(1-\beta)$.

From the previous analysis, we conclude that there exists a Pareto Optimal

perfect equilibrium of this form iff

$$(36) \qquad \ln(2-\beta) - \frac{1-\delta}{\delta}\ln(1+\beta) \geq (1-\alpha)\ln 2$$

In the non-symmetric case, the Pareto-optimal total extractions are sensitive to the welfare weights: in other words, it is no longer true that all Pareto Optimal extraction levels are the same. Solving the general welfare problem, we can see that (h(1),h(2)) solves (32) iff

$$(37) \qquad h(1) = w(1-\beta_1) \qquad\qquad h(2) = (1-w)(1-\beta_2)$$

IV. The Exhaustible Resource Case

An interesting special case is that of a non-renewable resource, obtained by setting a = 1. We further limit attention to the symmetric case.

Conditions (13) for a perfect equilibrium of the form we have been studying simplify to

$$(38) \quad (1-\delta)\ln\left[\frac{h(1)}{1-h+h(1)}\right] + \delta\ln(1-h) \geq (1-\delta)\ln(1-\delta) + \delta\ln\left[\frac{\delta}{2-\delta}\right]$$

$$(38) \quad (1-\delta)\ln\left[\frac{h-h(1)}{1-h(1)}\right] + \delta\ln(1-h) \geq (1-\delta)\ln(1-\delta) + \delta\ln\left[\frac{\delta}{2-\delta}\right]$$

If we insert the intersection value h(1) = h/2, we obtain the condition for h to be sustainable:

$$(40) \quad (1-\delta)\ln\left[\frac{h}{2-h}\right] + \delta\ln(1-h) \geq (1-\delta)\ln(1-\delta) + \delta\ln\left[\frac{\delta}{2-\delta}\right]$$

Now consider the case $\delta = 1/2$. Equation 40 is quadratic, and gives:

$$(41) \qquad 1/2 \leq h \leq 2/3$$

We can solve for the equilibrium shares h(1) and h(2) using (38) and (39). This gives two equations relating the shares:

$$(42) \qquad (1-6h(1))h(2) \geq 1 - 6h(1)(1-h(1))$$

$$(42) \qquad (1-6h(2))h(1) \geq 1 - 6h(2)(1-h(2))$$

These shares are shown in Figure 10 below.

A slightly more complicated case occurs at $\delta = 2/3$: in this case, (40) is equivalent to the cubic equation:

(43) $12h^3 - 24h^2 + 13h - 2 \geq 0$

and the sustainable total extractions satisfy:

(44) $0.2713 \leq h \leq 0.5000$

The equations relating the shares of the two players are:

(45) $12h(i)[1-h(1)-h(2)]^2 + h(j) - 1 \geq 0 \quad \text{for } i \neq j \in \{1,2\}$

What is interesting about these two examples is the fact that increasing myopia increases the sustainable levels of extraction. In one sense, this is what we would expect by analogy with the repeated game results: as the players get less myopic, the results are "nicer": at any $\delta = 2/3$ perfect equilibrium of this form, the time path of stocks dominates that at any $\delta = 1/2$ equilibrium. In another sense, the result is surprising, since increases in the discount factor for repeated games are associated with increases in the set of equilibrium outcomes, whereas this example seems to suggest a shift in the set.

The impression that something different is going on is reinforced if we compute the comparative statics results near the two discount factors:

	$\delta = 2/3$		$\delta = 1/2$	
	minimum	maximum	minimum	maximum
h	.2713	.5	.5	.6667
G_h	.5067	-.4444	.3333	-.3750
G_δ	.6299	-.5	.4776	-.3333
h_δ	-1.2431	-1.1251	-1.4328	-0.8889

At both points, small increases in the discount factor are associated with decreases in both the maximum and minimum sustainable rates of extraction, with the latter falling faster than the former.

Of course, to say that the stock is being exhausted at a slower rate does not really mean that the players are "more cooperative." In order to make such a statement, we would have to assert something about the optimality of the sustainable extraction rates. Even a monopolist will depelete stocks at a slower rate, the higher is his discount factor.

There is a Pareto Optimal perfect equilibrium of the form we have been studying if

(46) $\delta \ln(2-\delta) \geq (1-\delta)\ln(1+\delta)$

which is to say $\delta \geq 1/2$. The shares at such an equilibrium satisfy

(47) $\ln(h(1)) - \ln(\delta+h(1)) \geq \ln(1-\delta) - [\delta/(1-\delta)]\ln(2-\delta) \equiv \ln(m)$

(48) $\ln(1-\delta-h(1)) - \ln(1-h(1)) \geq \ln(m)$

So the equilibrium shares satisfy

(49) $$\frac{1-\delta-m}{1-m} \geq h(1) \geq \frac{m\delta}{1-m}$$

This interval is nonempty as long as $\delta \geq 1/2$, and may well be nontrivial. For example, if $\delta = .75$, we get $h(1) \in [.1101,.1399]$. Therefore, even in the simple case of a nonrenewable resource extracted by identical duopolists at the Pareto Optimal rate, there is still room for a range of perfect equilibrium outcomes.

We shall close this discussion of the exhaustible case by describing the outcomes in utility terms, using (6). Neglecting the utility of the initial stock, the utility of player i is proportional to:

$$(1-\delta)\ln(h(i)) + \delta\ln(1-h(1)-h(2))$$

At the maximal and minimal rates of extraction, the players each consume the same proportion of the existing stock. If total extraction is h, utility is therefore proportional to:

$$(1-\delta)\ln(h) + \delta\ln(1-h)$$

However, it is not necessarily the case that extremes of total extraction are associated with extreme points of utility. In fact, the only case where this does happen is at the point $\delta = 1/2$, where there is a unique Pareto Optimal perfect equilibrium corresponding to the slowest perfect equilibrium rate of extraction. In all other cases, the utilities associated with perfect equilibria are a "folded" image of the corresponding set of perfect equilibrium extraction rates.

To show this, Figures 10 and 11 present the set of sustainable perfect equilibrium shares and the utilities associated with them for $\delta = 1/2$ and 2/3.

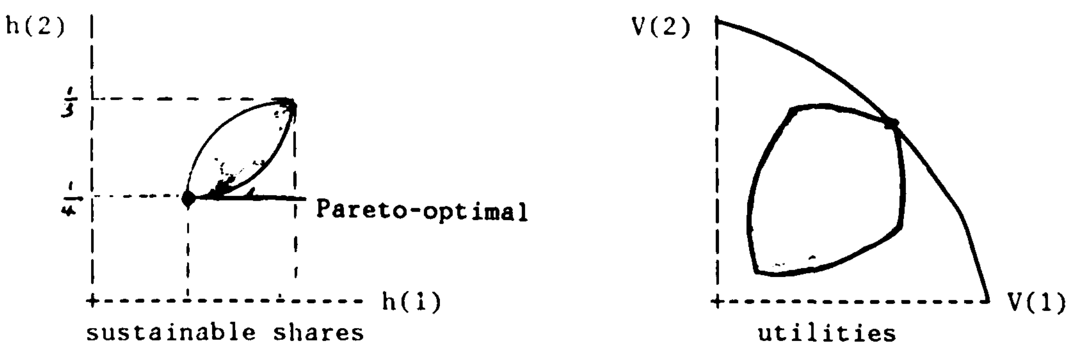

Figure 10: $\delta = 1/2$

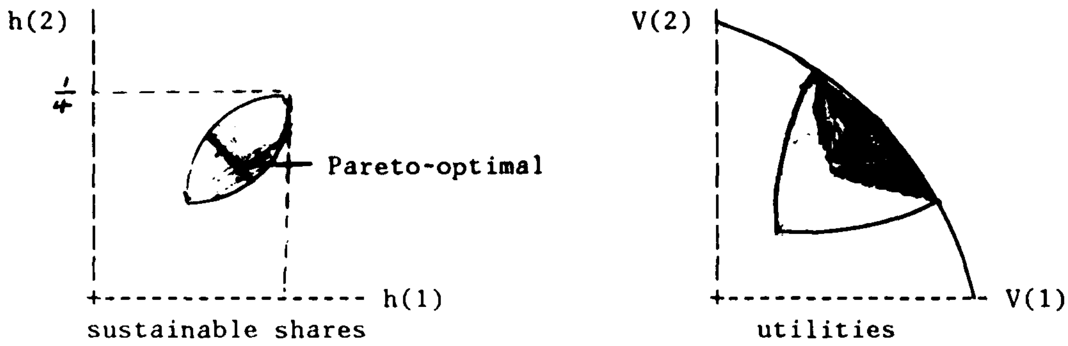

Figure 11: $\delta = 3/4$

The folding occurs at greater discount factors because higher total rates of extraction are associated with more rapid exhaustion of the resource, and thus with lower total utility. However, they are still perfect equilibrium outcomes.

To determine the size of the fold, or the range of Pareto Optimal utilities associated with perfect equilibria, we fix the discount factor, and thus total extraction, and vary the individual shares using the perfect equilibrium conditions. The following table gives the maximal and minimal Pareto Optimal utilities for a player at various discount rates. The set of utilities is symmetric, so we give only one set.

δ	h^*	min h(1)	max h(1)	min V(1)	max V(1)
.5	.5	.25	.25	-2.0794	-2.0794
.55	.45	.22	.23	-2.2005	-2.2446
.6	.4	.191	.209	-2.3317	-2.4217
.65	.35	.163	.187	-2.4765	-2.6143
2/3	1/3	.1538	.1795	-2.5826	-2.6827
.7	.3	.1360	.1640	-2.6399	-2.8276
.75	.25	.1101	.1399	-2.8298	-3.0695
.8	.2	.0854	.1146	-3.0589	-3.3530
.85	.15	.0620	.0880	-3.3509	-3.7022
.9	.1	.0399	.0601	-3.7593	-4.1706
.95	.05	.0192	.0308	-4.4541	-4.9286
.99	.01	.0037	.0063	-6.0939	-6.5915

Unlike the repeated-game situation, the normalized utilities are not directly comparable, as can be seen in the fact that the more far-sighted agents are worse off at Pareto Optimal equilibria. The rates of extraction are probably a better standard of comparison.

V. Discussion

This paper has shown that the use of threat strategies in a dynamic game

can widen the possibilities for equilibrium behavior, even when the threats are required to be credible. In essence, the players in such a game have a great deal of leeway for threatening each other: to each possible "cooperative" choice of extraction rates there corresponds a unique sequence of stocks, given by (3). In seeing whether this behavior can be made to result from an equilibrium, players are allowed to choose their extraction levels freely at any other stocks. If this is regarded as a game in extensive form, the players are allowed to take the period at which they observe a particular stock into account, and react accordingly. That is to say, the strategies we are considering here are explicitly "closed-loop" strategies. This is in contrast to the stationary strategies considered by Mirman [1979], which have the property that current consumption depends only on the current stock.

It is possible to construct threat strategies that depend only on current stocks, but the threats would be empty, since players would be unable to tell whether a given stock level resulted from defections or from variations in the initial stock. Threats could be made somewhat more powerful if allowed to depend on both initial and current stocks, but they would still fail to deter defections of a certain kind: suppose that along the cooperative path stocks were to increase monotonically to the steady state. A player could unilaterally increase his consumption by an amount sufficient to ensure that the stock in the next period would be one of those that had already occurred. The other player, if using an open-loop strategy, would be unable to detect such a defection, and would proceed as if nothing had happened.

Such partial approaches seem unsatisfactory, so we have chosen to use time- and state-dependant strategies. However, this game is played under conditions of certainty: the growth law is deterministic, so that a player knowing the sequence of his own past consumptions and of the stocks will also know the past consumption by his opponent. introducing noise into the system would complicate matters. In the first place, even if the players could observe each other's catch the optimal policies would not be stationary. In addition, a fluctuation in the stock observed by a player could result from either natural or unnatural causes, and the posterior probability that a particular stock reflects deviation from cooperative behavior is not well-defined if the players are following pure strategies. The alternative is to use a concept similar to trembling-hand perfectness or sequentiality [Kreps and Wilson, 1982].

In the deterministic case, we have characterized a set of subgame-perfect equilibria. The results would have been the same had we used the trembling-hand perfectness concept, since each player's payoff depends only on the aggregate of the other players' extractions. Since the Cournot-Nash policies are undominated in this game, the strategies used in Section III are also trembling-hand perfect.

Finally, there are other strategies that can be used to construct subgame-perfect equilibria. For instance, any perfect equilibrium path can be used as a threat if a particular path is defected from. In other words, we

could choose that path among those in Section III with the lowest payoff for player 1, and threaten to play according to this rule if player 1 defects, and similarly for player 2. In the repeated-game situation this does not add to the set of outcomes, but it may do so in the dynamic game. In fact, to any pair of extraction rates h = (h(1),h(2)) we may associate "sufficient punishments", which are other extraction rates offering one or the other player a lower payoff. Let the set of sufficient punishments for player i be denoted P(i,h), and the intersection P(h). Any h for which P(h) is nonempty is an equilibrium h. Finding a sequence h* = h(1),...,h(t),... such that h(t) belongs to P(h(t-1)) for all t, is necessary and sufficient for h* to be the outcome of a perfect equilibrium. In this paper, we have limited ourselves to those h* for which the Cournot-Nash policies belong to P(h*), since the latter are fixed points of the correspondance P.

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